Printed page: 4 Subject Code: BAS0204

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NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

B. Tech

SEM: II THEORY EXAMINATION (2024-2025)

Subject: Mathematical Foundations - II

Time: 3 Hours **Max. Marks:100**

General Instructions:

IMP: Verify that you have received question paper with correct course, code, branch etc.

1. *This Question paper comprises of three Sections -A, B, & C. It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.*
2. *Maximum marks for each question are indicated on right hand side of each question.*
3. *Illustrate your answers with neat sketches wherever necessary.*
4. *Assume suitable data if necessary.*
5. *Preferably, write the answers in sequential order.*
6. *No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.*

SECTION – A 20

1. Attempt all parts:-

1-a. Beta function $\beta(3,8) = a$. Then (CO1,K1) 1

(a) $a = \frac{1}{720}$

(b) $a = \frac{1}{360}$

(c) $a = \frac{1}{180}$

(d) $a = \frac{3}{8}$

1-b. If $I = \int_0^1 \int_0^x sinx\ dydx$. Then (CO1,K2) 1

(a) $I = sinx - cosx$

(b) $I = sin1 - cos1$

(c) $I = cos1 - sin1$

(d) $None\ of\ the\ above$

1-c. The complementary function of $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$ is (CO2,K1) 1

(a) $c_1 e^x + c_2\, e^{-x}$

(b) $e^{-\frac{1}{2}x}\left(c_1 cos\frac{\sqrt{3}}{2}x + c_2 sin\frac{\sqrt{3}}{2}x\right)$

(c) $c_1 cosx + c_2 sinx$

(d) $e^{\frac{1}{2}x}\left(c_1 cos\frac{\sqrt{3}}{2}x + c_2 sin\frac{\sqrt{3}}{2}x\right)$

1-d. The particular integral of $(D^2 + D + 2)y = e^{-2x}$ is (CO2,K2) 1

(a) $-\frac{e^x}{4}$

(b) $\frac{e^x}{4}$

(c) $-\frac{e^{2x}}{4}$

(d) $\frac{e^{-2x}}{4}$

1-e. The relation $z = (x + a)(y + b)$, where a and b are constants , form the linear partial differential equation is (CO3,K2) 1

(a) $p + q = z$

(b) $pq = 0$

(c) $pq = z$

(d) None of these

1-f. PDE: $b\frac{\partial u}{\partial t} + a\frac{\partial^2 u}{\partial x^2} = 0$, a and b are positive even constant. The given PDE is (CO3,K2) 1

(a) Hyperbolic

(b) Elliptic

(c) Parabolic

(d) None of these

1-g. If $L(sin^2 t) = F(s)$, then (CO4,K1) 1

(a) $F(s) = \frac{2}{s^2+4}$

(b) $F(s) = \frac{1}{2}\left(\frac{1}{s} - \frac{s}{s^2+4}\right)$

(c) $F(s) = \frac{1}{2}\left(\frac{1}{s} - \frac{s}{s^2-4}\right)$

(d) $Can\ not\ evalute$

1-h. Inverse Laplace transform of $\frac{b}{(s-a)^2-b^2}$ is (CO4,K1) 1

(a) $e^{at} \sinh bt$

(b) $e^{at} sin\, b\, t$

(c) $e^{at} \cosh bt$

(d) $e^{at} cos\, b\, t$

1-i. A sum of money amounts to three times itself in 6 years at compound interest. In how many years will the same amount become nine times itself? (CO5,K2) 1

(a) 10 years

(b) 12 years

(c) 15 years

(d) 18 years

1-j. Ramesh is the father of Amit. Amit is the brother of Priya. Priya is the mother of Rahul. How is Ramesh related to Rahul? (CO5,K2) 1

(a) Uncle

(b) Brother

(c) Grandfather

(d) Father

2. Attempt all parts:-

2.a. Evaluate $\int_0^1 x^8(1-x)^9\,dx$ (CO1,K2) 2

2.b. Solve: $\frac{d^2y}{dx^2} - 4y = x^2$ (CO2,K2) 2

2.c. Solve: $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = e^{x+2y}$ (CO3,K2) 2

2.d. Find the Laplace transform of $\frac{e^{-t}\sin t}{t}$ (CO4,K3) 2

2.e. A boy has coins in the denominations of Rs.1 and Rs.2. If he has total 30 coins and the value of coins is Rs.48. Find the number of Rs.1 coins he has. (CO5,K2) 2

SECTION – B 30

3. Answer any <u>five</u> of the following-

3-a. Show that $\beta(p,q) = \int_0^1 \frac{x^{p-1}+x^{q-1}}{(1+x)^{p+q}}\,dx$ (CO1,K3) 6

3-b. Find the volume of the cylindrical column standing on the area common to the parabolas $x = y^2, y = x^2$ as a base and cut-off by the surface $z = 12 + y - x^2$. (CO1,K3) 6

3-c. By reducing to homogeneous, solve the differential equation $(1+x)^2\frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 4\cos\{\log(1+x)\}$. (CO2,K2) 6

3-d. Solve: $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 8x^2e^{2x}\sin 2x$ (CO2,K2) 6

3-e. Solve the linear partial differential equation $\left(D^2 - DD' - 2D'^2 + 2D + 2D'\right)z = \sin(2x+3y)$ (CO3,K3) 6

3-f. Evaluate: $\int_0^\infty t^3e^{-t}\sin t\,dt$ (CO4,K3) 6

3-g. The ratio of Sara's age 4 years ago and Vashali's age after 4 years is 1:1.Presently, the ratio of their age is 5:3. Find the ratio between Sara's age 4 years hence and Vashali's age 4 years ago. (CO5,K2) 6

SECTION – C 50

4. Answer any <u>one</u> of the following-

4-a. Change the order of integration in $I = \int_0^1 \int_{y^2}^{2-y} xy\, dxdy$ and hence evaluate the same. (CO1,K3) 10

4-b. The plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ meets the axes in A,B and C. Apply Dirichlet's integral to find the volume of the tetrahedron OABC. Also find its mass if the density at any point is $kxyz$. (CO1,K3) 10

5. Answer any <u>one</u> of the following-

5-a. Solve by the method of variation of parameters: $\frac{d^2y}{dx^2} + a^2y = \sec ax.$ (CO2,K2) 10

5-b. Solve the following simultaneous differential equation $\frac{dx}{dt} + 5x - 2y = t, \frac{dy}{dt} + 2x + y = 0$, given that $x = y = 0$ when $t = 0$ (CO2,K2) 10

6. Answer any <u>one</u> of the following-

6-a. Solve: $(y - z)p + (x - y)q = (z - x).$ (CO3,K3) 10

6-b. Solve the linear partial differential equation $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = y \sin x$ (CO3,K2) 10

7. Answer any <u>one</u> of the following-

7-a. State convolution theorem and hence evaluate $L^{-1}\left\{\frac{s}{(s^2+1)(s^2+4)}\right\}$ (CO4,K3) 10

7-b. Using Laplace transform, find the solution of the initial value problem $\frac{d^2y}{dt^2} + 9y = 6\cos 3t\,; y(0) = 2, y'(0) = 0.$ (CO4,K2) 10

8. Answer any <u>one</u> of the following-

8-a. (i) Two equal vessels are filled with the mixture of water and milk in the ratio of 3:4 and 5:3 respectively. If the mixture are poured into a third vessel. Find the ratio of water and milk in the third vessel.
(ii) Varun drove his car for 80 kms due North. Then he turned left and drove for 100 kms. Again he turned left & drove yet another 80 kms. Again he turned left and drove his car 120 kms. How far do you think he actually drove his car from the initial position? 10

(CO5,K2)

8-b. (i) Sahil's capital is 1/6 times more than Chaya's capital. Chaya invested her capital at 20 % per annum for 2 years (compounded annually). At what rate % p.a. simple interest should Sahil invest his capital so that after 2 years, they both have the same amount of capital. 10

(ii) A, B and C started a business each investing Rs.10000. After 4 months A withdraws Rs.3000, B withdraws Rs.4000, C invest Rs.3000 more. At the end of the years, a total profit was Rs.32800. Find the share of C. (CO5,K2)